

TWO-PHOTON EXCHANGE IN ELECTRON DEUTERON SCATTERING

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Abstract

It is shown that the amplitude of elastic ed scattering beyond Born approximation contains six generalized form factors, but only three linearly independent combinations of them (generalized charge, quadrupole and magnetic form factors) contribute to the reaction cross section in the second order perturbation theory. We examine two-photon exchange and find that it includes two types of diagrams, when two virtual photons interact with the same nucleon and when the photons interact with different nucleons. We discuss contribution of the two-photon exchange in reaction observables, generalized \mathcal{A} and \mathcal{B} structure functions and tensor polarization of the deuteron.

1 Introduction

A study of electromagnetic structure of the deuteron, the simplest nucleon system, provides with important information about nucleon-nucleon interaction. Because the deuteron is a spin-1 system its electromagnetic current characterized by three form factors, charge G_C , quadrupole G_Q and magnetic G_M form factors. Due to smallness of the fine structure constant α the form factors are usually extracted from experimentally measurable observables in the framework of Born approximation (one-photon exchange, OPE). Nevertheless theoretical calculations [1, 2, 3, 4] show that effects beyond OPE may significantly change results of such procedure.

In what follows we calculate amplitude of two-photon exchange (TPE) of the elastic ed scattering, one of the mostly important effects beyond OPE, and estimate TPE contribution in observables of the process.

2 Observables beyond one-photon exchange

From P and T invariance it follows that elastic scattering amplitude of a spin- $\frac{1}{2}$ particle (electron) on a spin-1 particle (deuteron) is determined by 12 invariant amplitudes. Putting the electron mass to zero reduces the number of the invariant amplitude (form factors) to 6 and the spin structure of the amplitude in the Breit frame may be specified by the following

parametrization:

$$T_{\lambda'\lambda;h} = \begin{pmatrix} \mathcal{G}_{11} \cos \frac{\theta}{2} & -\sqrt{\frac{\eta}{2}} \mathcal{G}_{10}^h & \mathcal{G}_{1,-1}^h \\ \sqrt{\frac{\eta}{2}} \mathcal{G}_{10}^{-h} & \mathcal{G}_{00} \cos \frac{\theta}{2} & -\sqrt{\frac{\eta}{2}} \mathcal{G}_{10}^h \\ \mathcal{G}_{1,-1}^{-h} & \sqrt{\frac{\eta}{2}} \mathcal{G}_{10}^{-h} & \mathcal{G}_{11} \cos \frac{\theta}{2} \end{pmatrix}. \quad (1)$$

Here $T_{\lambda'\lambda;h}$ is reduced amplitude, which is connected with the usual amplitude by

$$\mathcal{M} = \frac{16\pi\alpha}{Q^2} E_e E_d T_{\lambda'\lambda;h}, \quad (2)$$

λ and λ' are spin projections of the deuteron and h is sign of electron helicity; E_e and E_d are electron and deuteron energies and θ is the scattering angle in the Breit system; $\eta = Q^2/4m_d^2$;

$$\mathcal{G}_{10}^h = f_1 + h \sin \frac{\theta}{2} f_2, \quad \mathcal{G}_{1,-1}^h = f_3 + h \sin \frac{\theta}{2} f_4. \quad (3)$$

The form factors \mathcal{G}_{11} , \mathcal{G}_{00} , f_1 , ..., f_4 are complex functions of the two independent kinematical variables, for example Q^2 and θ .

In Ref. [4] instead of the form factors \mathcal{G}_{11} , \mathcal{G}_{00} , f_1 , ..., f_4 the following their linear combinations were introduced

$$\begin{aligned} \mathcal{G}_{11} &= \mathcal{G}_C - \frac{2}{3}\eta\mathcal{G}_Q, & \mathcal{G}_{00} &= \mathcal{G}_C + \frac{4}{3}\eta\mathcal{G}_Q, \\ f_1 &= \mathcal{G}_M + g_1 \sin^2 \frac{\theta}{2}, & f_2 &= \mathcal{G}_M - g_1, \\ f_3 &= g_2, & f_4 &= g_3. \end{aligned} \quad (4)$$

We call $\mathcal{G}_Q(Q^2, \theta)$ and $\mathcal{G}_M(Q^2, \theta)$ the generalized electric, quadrupole and magnetic form factors.

By standard calculations one derives the differential cross section and components of tensor polarization of the deuteron

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \sigma_M \mathcal{S}, \\ t_{20} &= \frac{-\eta}{3\sqrt{2}} \frac{[8(\Re \mathcal{G}_C^* \mathcal{G}_Q + \frac{\eta}{3} |\mathcal{G}_Q|^2) + |\mathcal{G}_M|^2 (1 + 2\text{tg}^2 \frac{\theta}{2})]}{\mathcal{S}}, \\ t_{21} &= \frac{\sqrt{\eta}}{\sqrt{3}\mathcal{S}} [-2 \cos \frac{\theta}{2} \eta \Re \mathcal{G}_M^* \mathcal{G}_Q - G_M \Re (\sin^2 \frac{\theta}{2} g_3 + g_2) - 2 \sin^2 \frac{\theta}{2} \cos \frac{\theta}{2} \eta \Re g_1 G_Q], \\ t_{22} &= \frac{-\cos^2 \frac{\theta}{2} \eta |\mathcal{G}_M|^2 - 4 \sin^2 \frac{\theta}{2} \eta G_M \Re g_1 + 4 \cos \frac{\theta}{2} (G_C - \frac{2}{3}\eta G_Q) \Re g_2}{2\sqrt{3} \cos^2 \frac{\theta}{2} \mathcal{S}}. \end{aligned} \quad (5)$$

In Eqs. (5) σ_M is the Mott cross section,

$$\begin{aligned} \mathcal{S} &= \mathcal{A} + \mathcal{B} \text{tg}^2 \left(\frac{1}{2} \theta_{\text{LAB}} \right), \\ \mathcal{A}(Q^2, \theta) &= |\mathcal{G}_C(Q^2, \theta)|^2 + \frac{8}{9} \eta^2 |\mathcal{G}_Q(Q^2, \theta)|^2 + \frac{2}{3} \eta |\mathcal{G}_M(Q^2, \theta)|^2, \\ \mathcal{B}(Q^2, \theta) &= \frac{4}{3} (1 + \eta) \eta |\mathcal{G}_M(Q^2, \theta)|^2 \end{aligned} \quad (6)$$

and $\mathcal{G}_K^* \mathcal{G}_L = G_K G_L + \delta \mathcal{G}_K^* \mathcal{G}_L + \mathcal{G}_K^* \delta \mathcal{G}_L$, $(K, L) = C, Q, M$.

The advantage of using the form factors \mathcal{G}_C , \mathcal{G}_Q and \mathcal{G}_M is that the expression for the cross section and t_{20} have the same form as in OPE approximation. Nevertheless the Rosenbluth separation of the structure functions $\mathcal{A}(Q^2, \theta)$ and $\mathcal{B}(Q^2, \theta)$ can no longer be done because they depend on two variables.

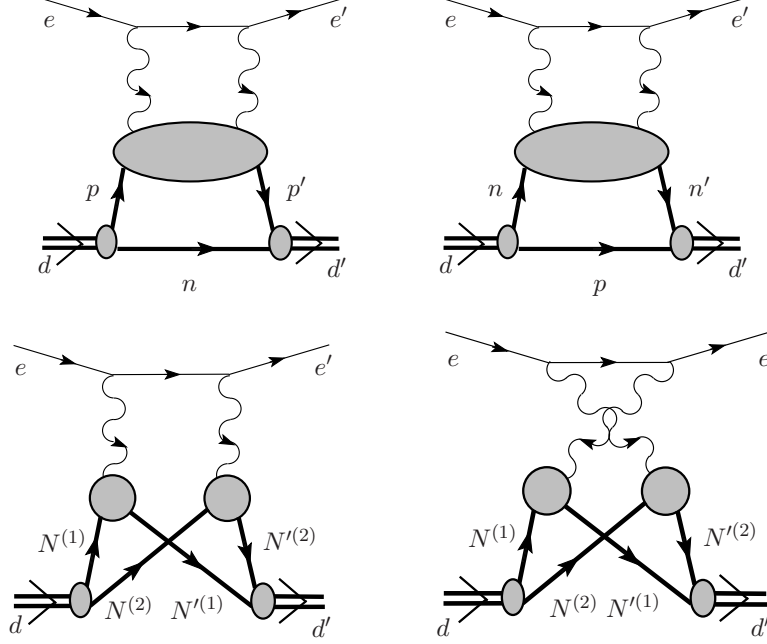


Figure 1: Two-photon exchange diagrams. The top diagrams correspond to the amplitudes \mathcal{M}_p^I and \mathcal{M}_n^I , the bottom diagrams to the amplitudes \mathcal{M}_p^{II} and \mathcal{M}_X^{II} .

3 Calculation of the two-photon exchange

In our calculation of TPE we consider two types of diagrams, where the virtual photons interact directly with the nucleons

$$\mathcal{M}_2 = \mathcal{M}^I + \mathcal{M}^{II}. \quad (7)$$

One of them, $\mathcal{M}^I = \mathcal{M}_p^I + \mathcal{M}_n^I$, corresponds to diagrams, where both photons interact with the same nucleon (Fig. 1, top). The other type, $\mathcal{M}^{II} = \mathcal{M}_p^{II} + \mathcal{M}_X^{II}$, corresponds to the diagrams, where the photons interact with different nucleons (Fig. 1, bottom).

Important input in calculation of \mathcal{M}^I is TPE amplitude for a nucleon N . It has the following structure [5]

$$\mathcal{M}_{2\gamma N} = \frac{4\pi\alpha}{Q^2} \bar{u}'_h \gamma_\mu u_h \left\langle \vec{p}'_N \sigma' \left| \hat{H}_N^\mu \right| \vec{p}_N \sigma \right\rangle, \quad (8)$$

where \hat{H}_N^μ is the “effective hadron current”

$$\hat{H}_N^\mu = \Delta \tilde{F}_1^N \gamma^\mu - \Delta \tilde{F}_2^N [\gamma^\mu, \gamma^\nu] \frac{q_\nu}{4m_N} + \tilde{F}_3^N K_\nu \gamma^\nu \frac{P^\mu}{m_N^2}. \quad (9)$$

In Eqs. (8) and (9) p_N and p'_N are the nucleon momenta, σ and σ' are the nucleon spin projections, $|\vec{p}_N \sigma\rangle$ and $|\vec{p}'_N \sigma'\rangle$ are the nucleon spinors, $K = (k + k')/2$, $P = (p_N + p'_N)/2$; $\Delta \tilde{F}_1^N$ and $\Delta \tilde{F}_2^N$ may be called corrections to the Dirac and Pauli form factors of nucleon N and \tilde{F}_3^N is a new form factor. All the quantities $\Delta \tilde{F}_1^N$, $\Delta \tilde{F}_2^N$ and \tilde{F}_3^N are of order α . They are complex functions of two kinematical variables, e.g. Q^2 and $\nu = 4PK$.

Considering the deuteron structure nonrelativistically one gets the following expressions

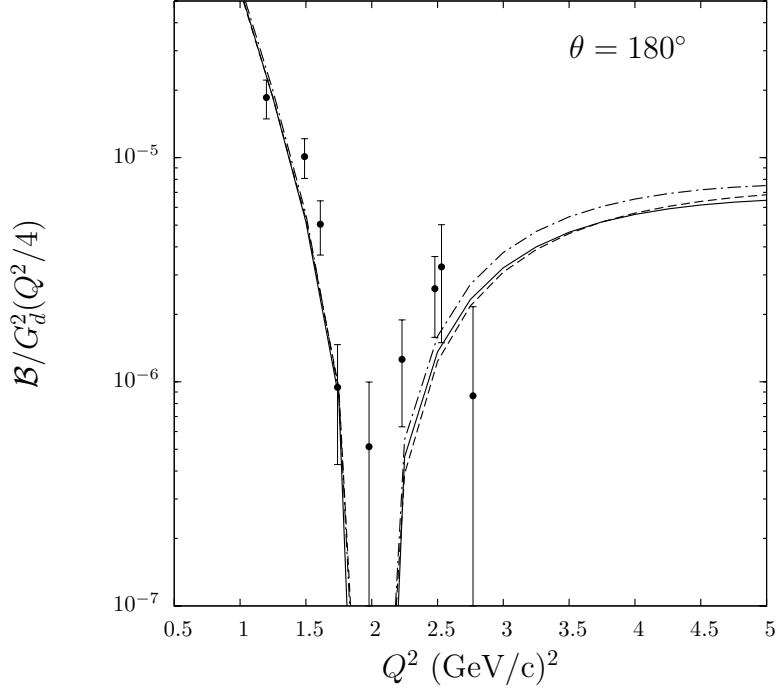


Figure 2: Results of calculations for \mathcal{B} at $\theta_{\text{LAB}} = 180^\circ$ (the curves are explained in the text). $G_d(t) = (1 + t/0.71)^{-2}$ is dipole form factor. Experimental data are from [7].

for appropriate TPE form factors for the elastic ed scattering [4]

$$\begin{aligned}\delta\mathcal{G}_C^{\text{I}} &= 2\delta\mathcal{G}_E^S [I_{00}^0(Q) + I_{22}^0(Q)], \quad \delta\mathcal{G}_Q^{\text{I}} = \frac{3\sqrt{2}}{\eta}\delta\mathcal{G}_E^S \left[I_{20}^2(Q) - \frac{1}{2\sqrt{2}}I_{22}^2(Q) \right], \\ \delta\mathcal{G}_M^{\text{I}} &= \frac{M}{m} \left\{ \frac{3}{2}\delta\mathcal{G}_E^S [I_{22}^0(Q) + I_{22}^2(Q)] + 2\delta\mathcal{G}_M^S \left[I_{00}^0(Q) - \frac{1}{2}I_{22}^0(Q) + \sqrt{\frac{1}{2}}I_{20}^2(Q) + \frac{1}{2}I_{22}^2(Q) \right] \right\}, \\ g_1^{\text{I}} &= -\epsilon \frac{E_e}{m} \mathcal{F}_3, \quad g_2^{\text{I}} = g_3^{\text{I}} = 0,\end{aligned}$$

where $\mathcal{F}_3 = 2\frac{M}{m}\tilde{F}_3^S \left[I_{00}^0(Q) - \frac{1}{2}I_{22}^0(Q) + \sqrt{\frac{1}{2}}I_{20}^2(Q) + \frac{1}{2}I_{22}^2(Q) \right]$, generalized nucleon electric and magnetic form factors are defined by (see Ref [6])

$$\delta\mathcal{G}_E^N = \Delta\tilde{F}_1^N - \tau\Delta\tilde{F}_2^N + \frac{\nu}{4m_N^2}\tilde{F}_3^N, \quad \delta\mathcal{G}_M^N = \Delta\tilde{F}_1^N + \Delta\tilde{F}_2^N + \frac{\epsilon\nu}{4m_N^2}\tilde{F}_3^N,$$

$\tau \approx 4\eta$, $\nu \approx m_N E_e$ and ϵ is the commonly used polarization parameter, $I_{\ell\ell}^L(Q) = \int_0^\infty dr j_L(\frac{1}{2}Qr) u_{\ell'}(r) u_\ell(r)$, $u_\ell(r)$ is the radial deuteron wave function for orbital momentum ℓ and $\delta\mathcal{G}_E^S = \frac{1}{2}(\delta\mathcal{G}_E^p + \delta\mathcal{G}_E^n)$, etc.

The amplitude \mathcal{M}^{II} was calculated within hard-photon approximation, i.e. assuming that each intermediate photon carries about half of the transferred momentum $\Delta_1 \sim \Delta_2 \sim \frac{q}{2}$. Omitting tedious calculations (see Ref. [4]), we obtain

$$\begin{aligned}\delta\mathcal{G}_C^{\text{II}} &= \kappa (G_{EE} - \frac{1}{3}\eta G_{MM}), \quad \delta\mathcal{G}_Q^{\text{II}} = -\frac{\kappa}{2}G_{MM}, \\ \delta\mathcal{G}_M^{\text{II}} &= \frac{2\kappa G_{EM}}{1 + \sin^2 \frac{\theta}{2}}, \quad g_1^{\text{II}} = \frac{\kappa G_{EM} \cos^2 \frac{\theta}{2}}{1 + \sin^2 \frac{\theta}{2}}, \quad g_2^{\text{II}} = g_3^{\text{II}} = \kappa\eta \cos \frac{\theta}{2} G_{MM}.\end{aligned}\tag{10}$$

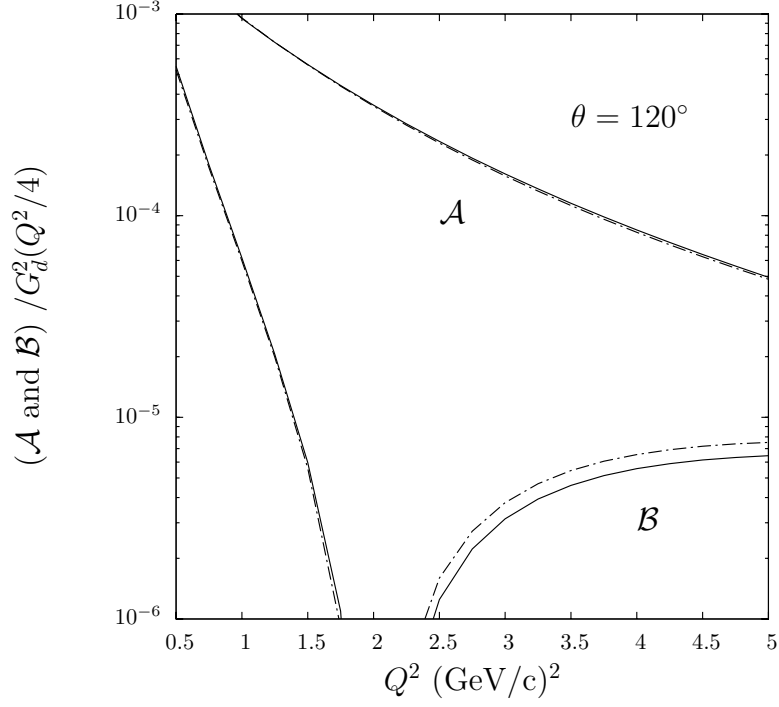


Figure 3: \mathcal{A} and \mathcal{B} at $\theta_{\text{LAB}} = 120^\circ$. Full curve for ONE+TPE (TPE is calculated with CD-Bonn deuteron wave function), dot-dashed for ONE approximation.

Here

$$\begin{aligned} G_{EE} &= G_E^p(\tfrac{1}{4}Q^2)G_E^n(\tfrac{1}{4}Q^2), \quad G_{MM} = G_M^p(\tfrac{1}{4}Q^2)G_M^n(\tfrac{1}{4}Q^2), \\ G_{EM} &= \tfrac{1}{2} [G_E^p(\tfrac{1}{4}Q^2)G_M^n(\tfrac{1}{4}Q^2) + G_M^p(\tfrac{1}{4}Q^2)G_E^n(\tfrac{1}{4}Q^2)] \end{aligned} \quad (11)$$

and

$$\kappa = -\frac{128\alpha E_e}{Q^4}\mathcal{C}, \quad \text{where} \quad \mathcal{C} = \frac{1}{(2\pi)^3} \int \frac{d^3p d^3p' U_0(p)U_0(p')}{1 + \frac{4E_e}{Qm_d}(p_z + p'_z) - 8\cos\frac{\theta}{2}\frac{E_e(p_x - p'_x)}{Q^2} + i0}. \quad (12)$$

In Eq. (12) $U_0(p)$ is S -component of the deuteron wave function in the momentum representation. To evaluate the integral above one can use the integral representation for the propagator $\frac{1}{\alpha + i0} = -i \int_0^\infty d\tau e^{i(\alpha + i0)\tau}$ and reduce \mathcal{C} to a one-dimensional integral

$$\mathcal{C} = -if \int_0^\infty \frac{dy}{y^2} e^{ify} u_0^2(y), \quad \text{where} \quad f = Q^2 \left[4E_e \sqrt{4\cos^2\frac{\theta}{2} + \frac{Q^2}{m_d^2}} \right]^{-1}. \quad (13)$$

4 Numerical calculations and conclusions

In Figure 2 we compare results of our calculations for \mathcal{B} at $\theta_{\text{LAB}} = 180^\circ$ with ONE analysis of Ref. [10] (dot-dashed). For TPE calculations we have used the deuteron wave functions for CD-Bonn and Paris potentials. ONE+TPE with TPE calculated with CD-Bonn deuteron [8] wave function and Paris [9] deuteron wave function are given by full and dashed curves, respectively. One sees that at $Q^2 > 2 \text{ GeV}^2$ TPE contribution becomes more than 10% in \mathcal{B} , while in \mathcal{A} it is not significant, see Figure 3.

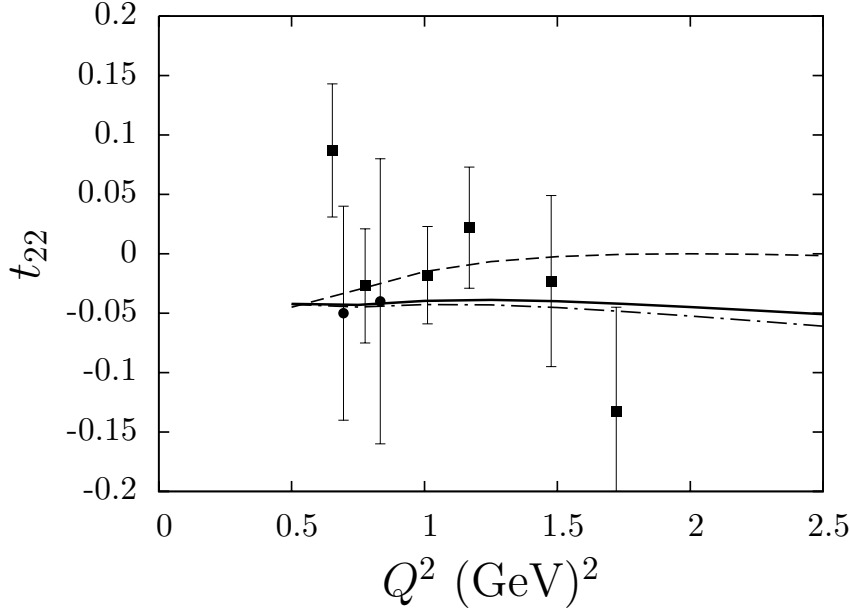


Figure 4: t_{22} at $\theta_{\text{LAB}} = 70^\circ$. Dashed line is for OPE-approximation, full and dashed-dot lines are for OPE+TPE with TPE calculated with CD-Bonn and Paris deuteron wave functions, respectively. Data are from [12] and [11] (circles and boxes, respectively).

TPE effect in t_{22} was found significant (Figure 4), but in t_{20} and t_{21} its contribution is minor ($<1\%$).

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